Concurrent Stochastic Lossy Channel Games

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joint work with **Muhammad Najib**[2], **Anthony W. Lin**[3,4], **Parosh Abdulla**[5]

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Outline

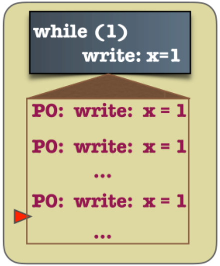
1. Lossy Channel Systems
2. Finite Concurrent Games
3. Infinite Concurrent Games

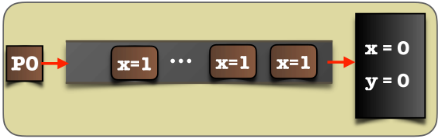
Channel Systems

Channel Systems (FIFO): Motivations

Modelisation and verification of systems with:

- Network transmissions;
- Transactional operations;
- TSO semantics [AABN18]





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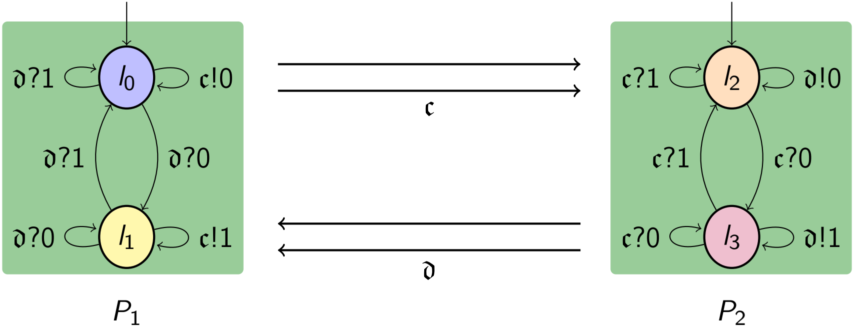

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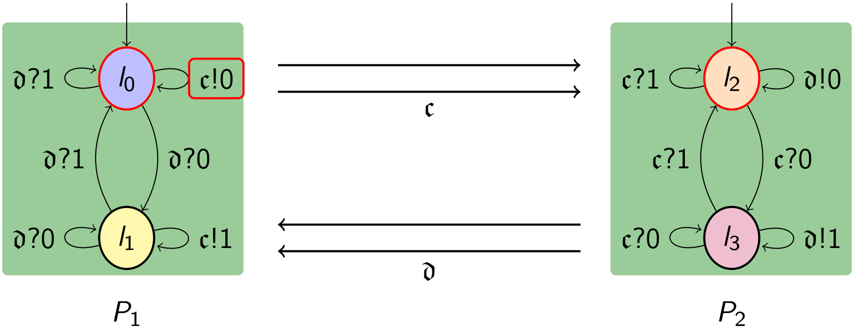

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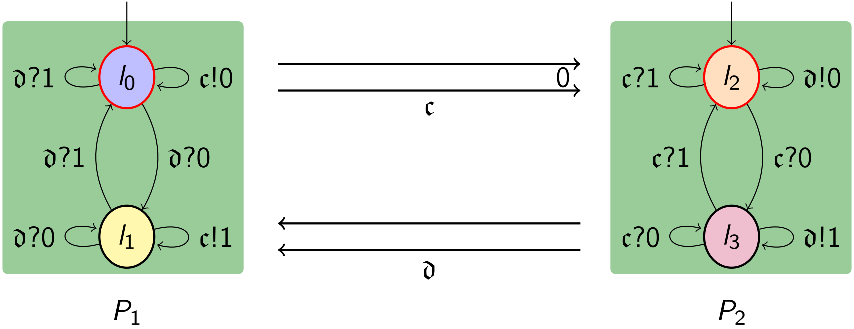

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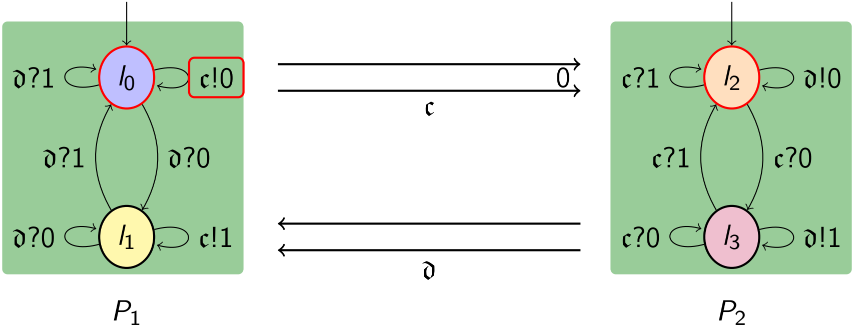

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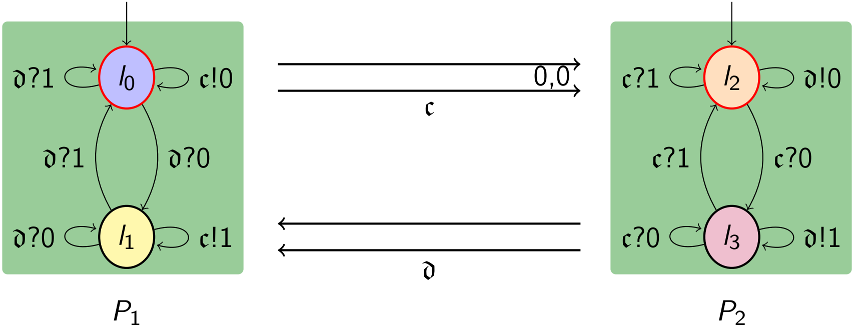

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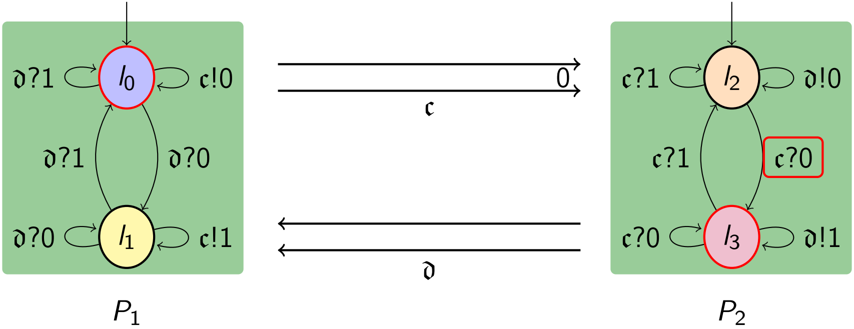

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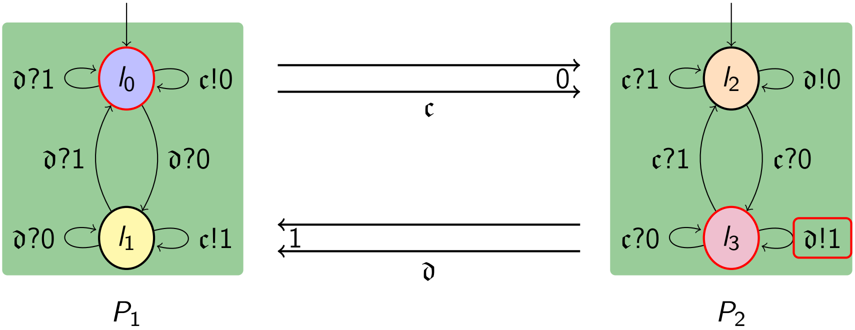

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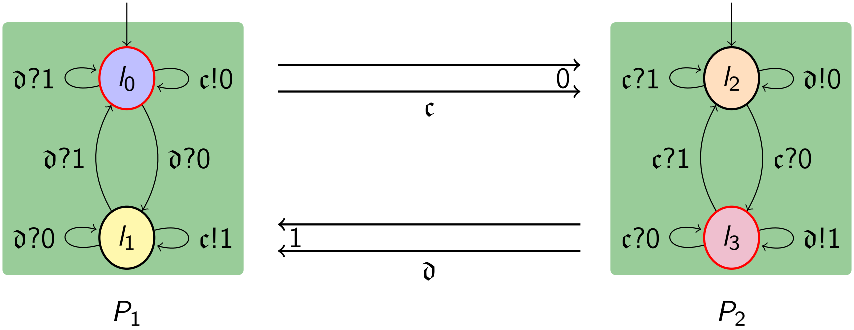

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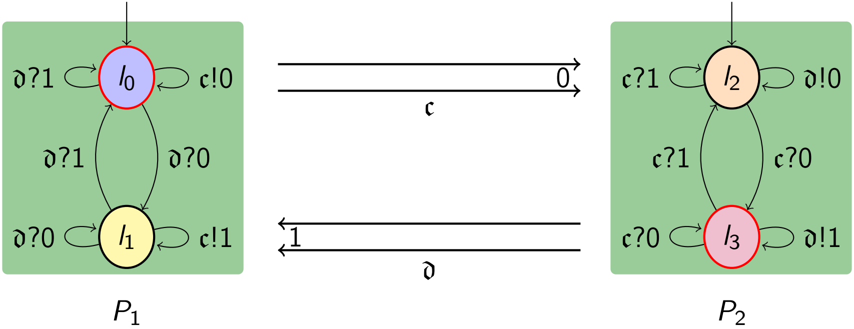

Communication:

- Send a message m on $\mathfrak{c}$: $\mathfrak{c}!m$
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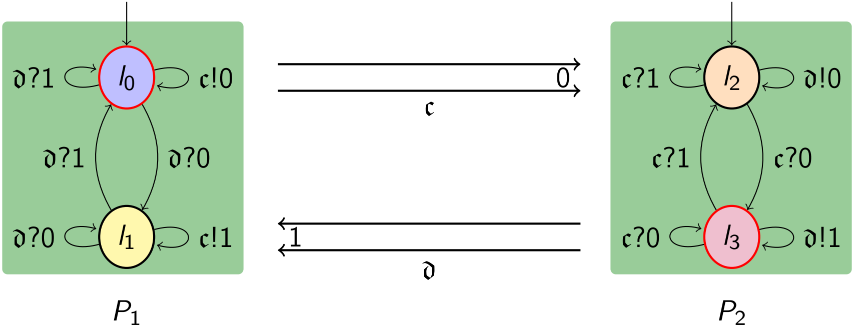

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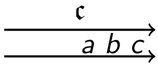
For this talk: only **one** channel, and **one** component.

Lossiness assumption

Assumption: at every round, every message *may* disappear.



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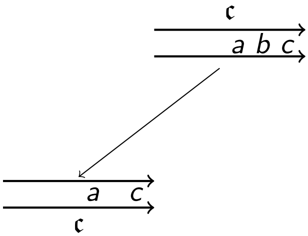

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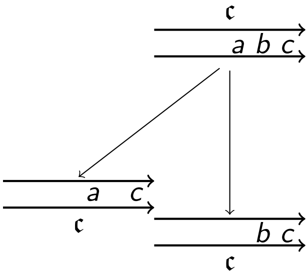

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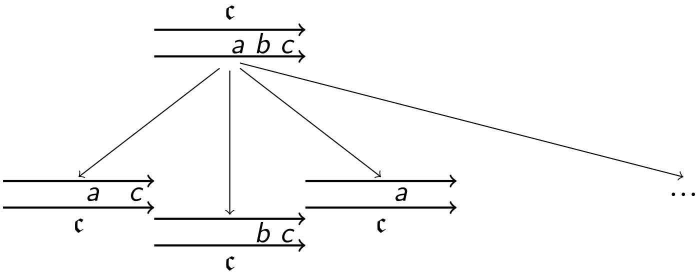

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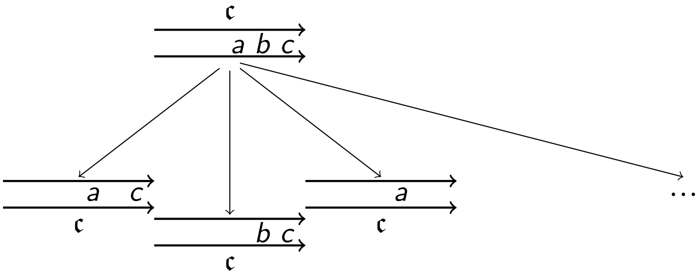

Effect of a transition $l \xrightarrow{f} l'$ on state $s = l \cdot w$:

1. Change location to l' ;
2. Apply the channel operation f on w to get w';
3. Drop from w' to $w'' \preceq w'$: subword ordering.

Result: $s' = l' \cdot w''$:

Subword-ordering $\preceq$ is a well-quasi order [FS01]

Definition ([FS01])

$(S, \preceq)$ is a well-quasi-order (WQO) if: $\forall (s_i)_{i \in \mathbb{N}} \in S^{\mathbb{N}}, \exists i < j : s_i \preceq s_j$

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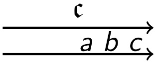
backward reachability scheme [FS01] for **non-deterministic schedulers**:

$$\bigcup_{n \geq 0} \mathrm{Pre}^n(R) = \{s \mid \exists (s_n) : s = s_0 \rightarrow s_1 \rightarrow \ldots s_k \in R\} = [\![\mathrm{E}(\Diamond R)]\!]$$

Lossiness in the probabilistic case

Stochastic case: the semantics is a Markov chain $(S, \Pr)$.

Local lossiness assumption: at every step, there is a positive probability $\lambda \in (0,1)$, that a letter in the channel is dropped. Every message drop event is **independent** from the others.



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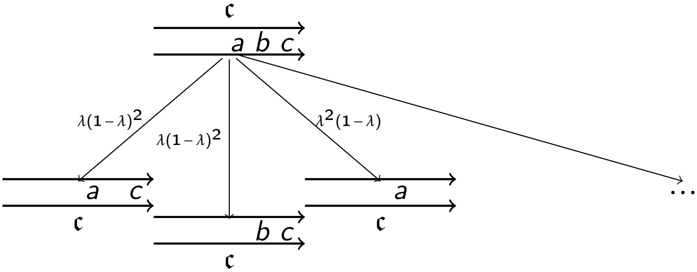

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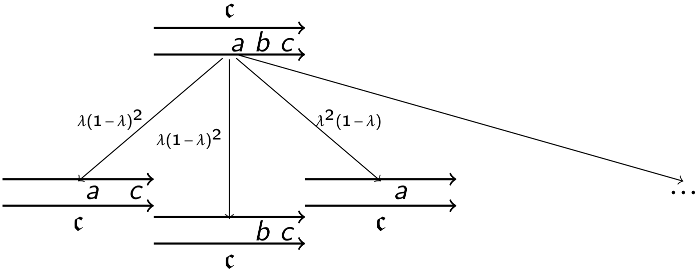
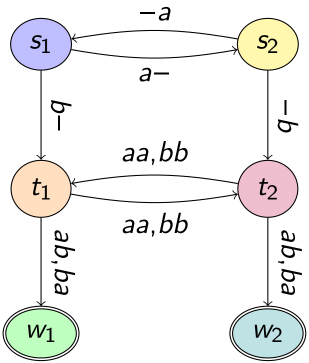


Qualitative setting:

$[\![\mathrm{NZ}(\Diamond R)]\!] = \{s \mid \mathrm{Pr}(s \rightarrow^* s' \in R) > 0\}$ $[\![\mathrm{AS}(\Diamond R)]\!] = \{s \mid \mathrm{Pr}(s \rightarrow^* s' \in R) = 1\}$

Stochastic Concurrent Finite Games

Concurrent Game on a Finite graph



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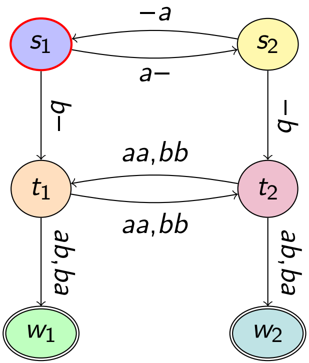

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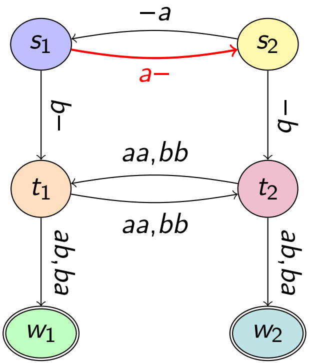

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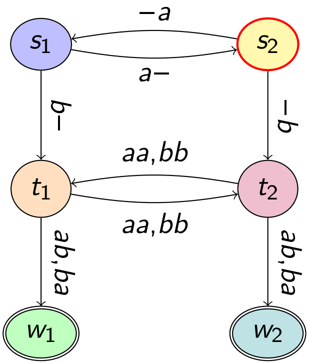

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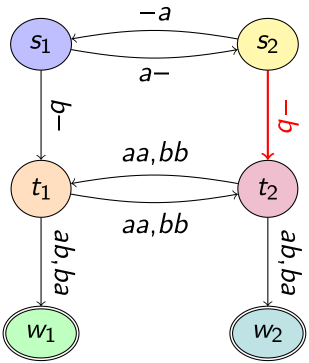

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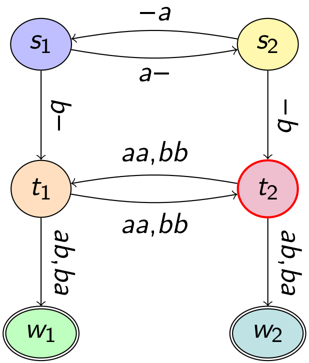

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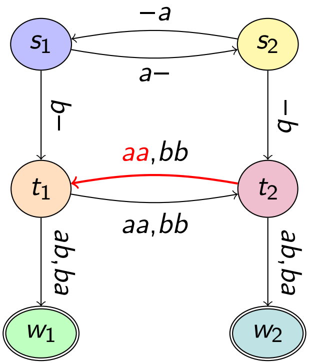

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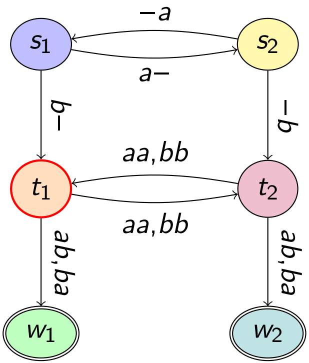

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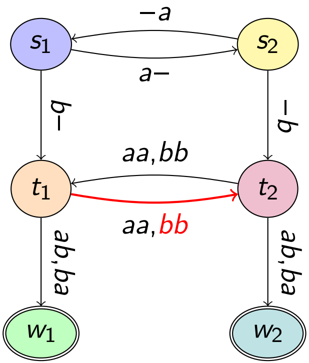

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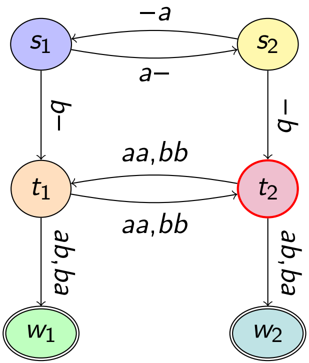

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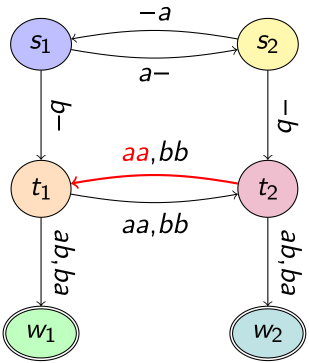

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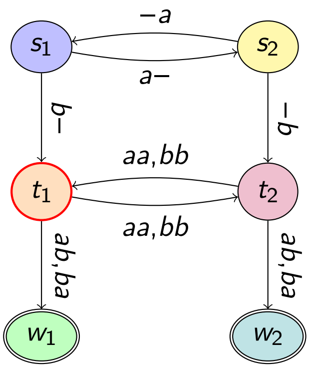

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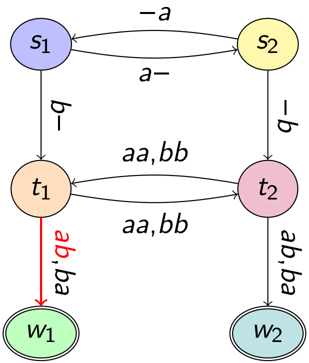

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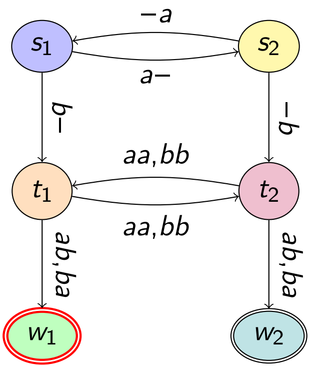

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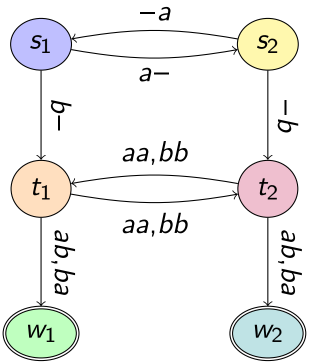

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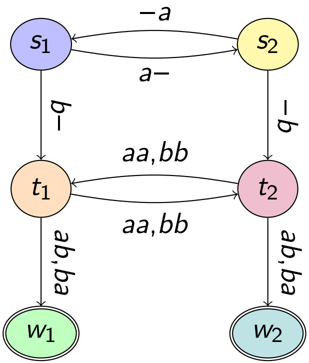

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- Simple Objectives: **Reachability**, **Safety**, **Büchi**, **CoBüchi**:

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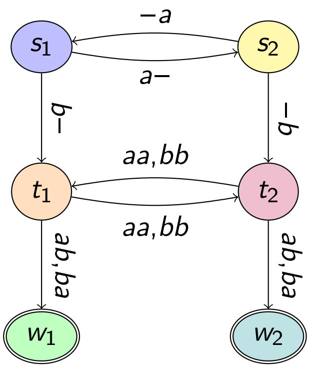

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  Ex: $\Diamond\, w_1$, $\Box\{w_2\}$, $\Box\Diamond\, s_1$, $\Diamond\Box\{t_1, t_2\}$
- Evaluated Qualitatively:
  almost surely, $\Pr(\ldots) = 1$ (AS)
  or **with positive probability** $\Pr(\ldots) > 0$ (NZ).

Different Ways of Winning

Players play strategies:

$$\forall i \in \mathrm{Agt},\ \sigma_i : \underbrace{s_0 \quad s_1 \ \ldots \ s_n}_{\text{history } \in S^+} \mapsto \delta \in \mathrm{Dist}(\underbrace{\mathrm{Act}_i(\ s_n \ )}_{\text{allowed actions in last state}})$$

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- **Zero-sum** case for two players, we compute the **winning regions**:
  $[\![\text{NZ}(\varphi_1)]\!]_1 = \{s \mid \exists \sigma_1 : \forall \sigma_2, \Pr^{\sigma_1,\sigma_2}(\varphi_1) > 0\}$
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  Does there exists $\vec{\sigma}$ in the core satisfying Γ?

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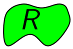
Fixed Point Algorithms for Concurrent Games [dAHK07]

Assume: Two players, **Zero-sum**, Reachability Objective for a given set $R \subseteq S$ of states.
How to compute the **winning set** for player 1?

Recipe for $[\![\mathrm{NZ}(\Diamond R)]\!]_1$

Take $X := R$



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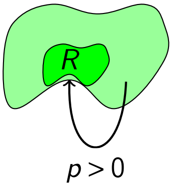

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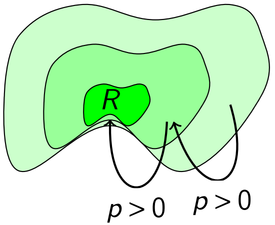

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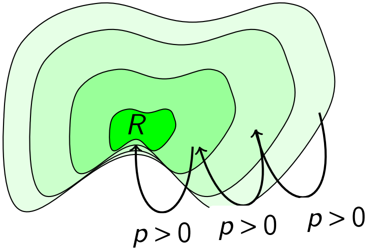

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Repeat until convergence:

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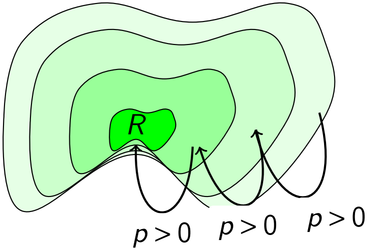
Fixed Point Algorithms for Concurrent Games [dAHK07]

Assume: Two players, **Zero-sum**, Reachability Objective for a given set $R \subseteq S$ of states.
How to compute the **winning set** for player 1?



Recipe for $[\![\mathrm{NZ}(\Diamond R)]\!]_1$

Take $X := R$

Repeat until convergence:

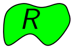
// Add to X any state that 1 **can enforce**
// reaching **with positive probability**
$X := X \cup \mathrm{Pre}_1(X)$

$\rightsquigarrow$ By **determinacy**, we can compute $[\![\mathrm{AS}(\Box R)]\!]_2$.

Fixed Point Algorithms for Concurrent Games [dAHK07] (bis)

How about **Almost-Sure Reachability**?

Recipe for $[\![\mathrm{AS}(\Diamond R)]\!]_1$



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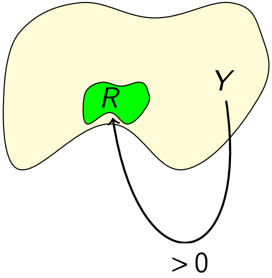

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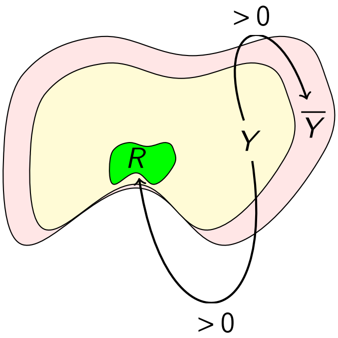

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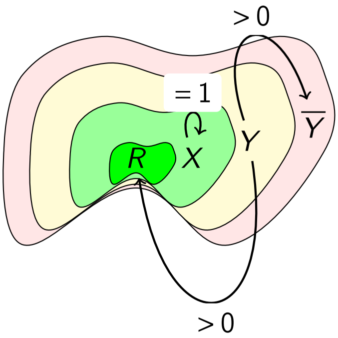

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// Now, **stay safe** in this set

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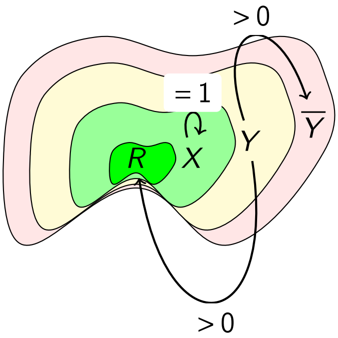

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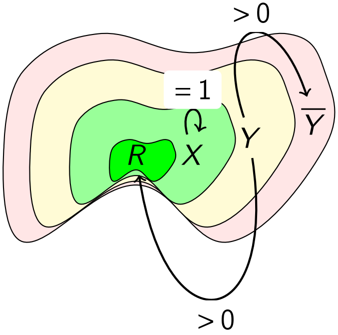

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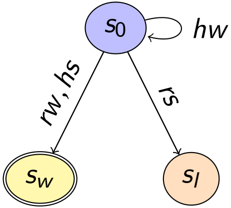
Example: Skirmish Game Analysis

- Step 1: $Y = X = \{\ s_0\ ,\ s_w\ \}$.



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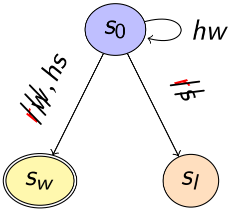

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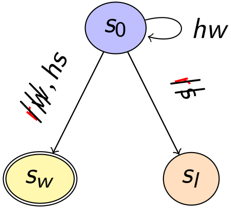

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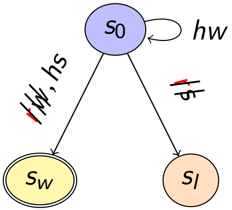

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- $\forall \epsilon > 0$; Player 1 can "win" with probability $1 - \epsilon$,
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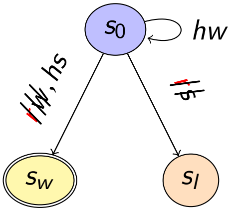

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$$\forall n, \sigma_2(\underbrace{s_0 \ldots s_0}_{n \text{ times}})[s] = \left(\frac{1}{2}\right)^{2^{-n}}$$

Concurrent Games + Lossy Channel Systems

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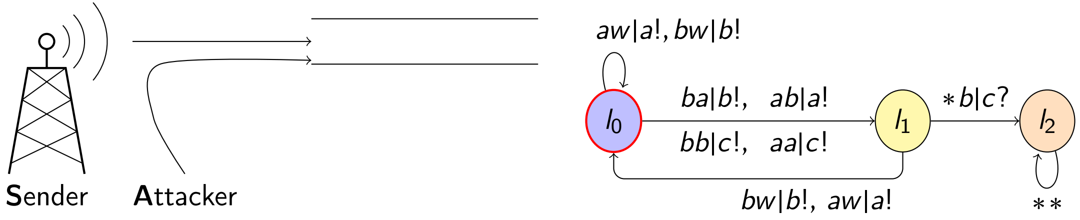
=

Infinite State Games

CSLCG: Def by Example



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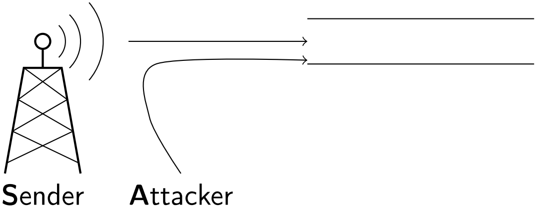

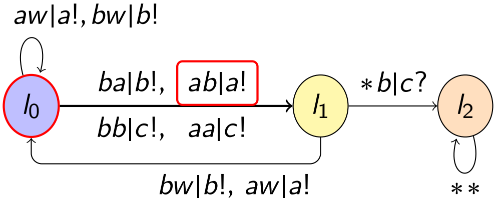

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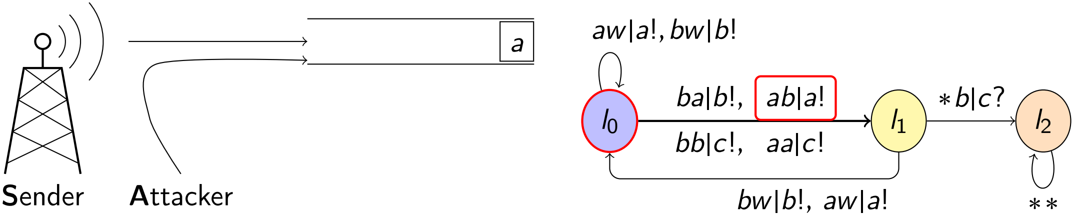

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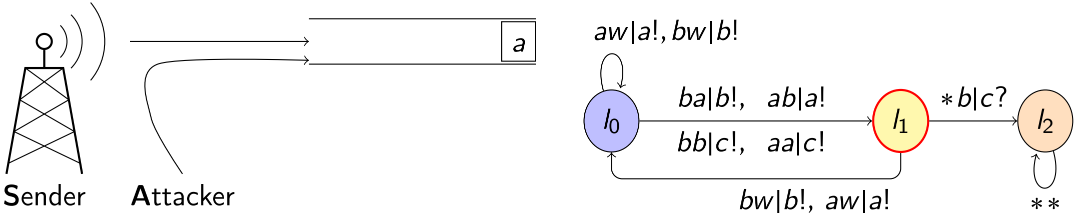

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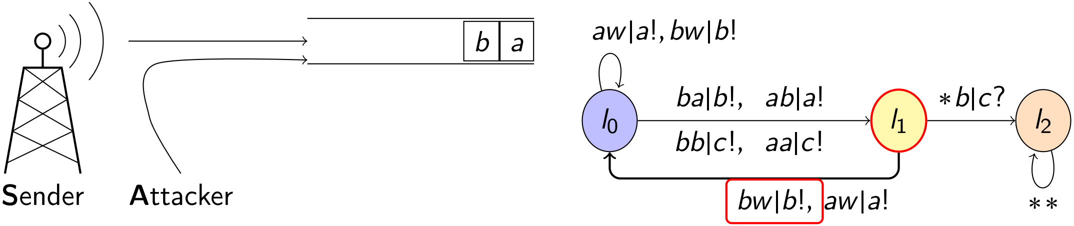

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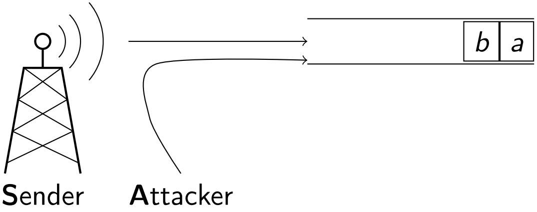

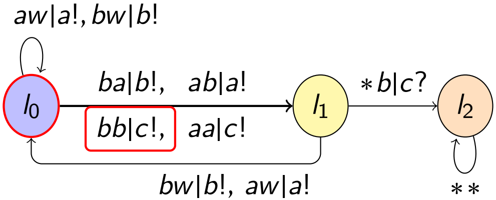

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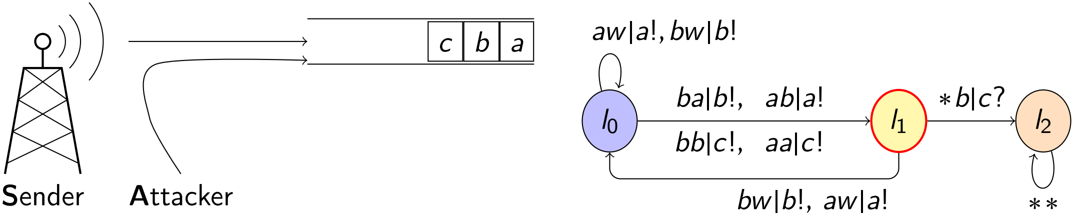

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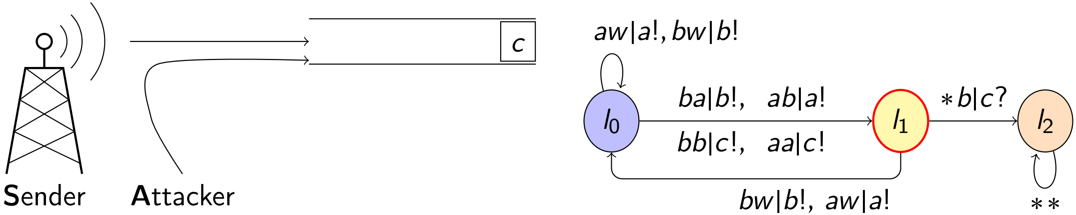

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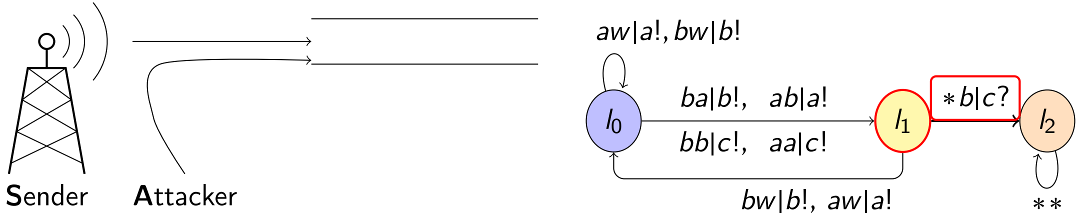

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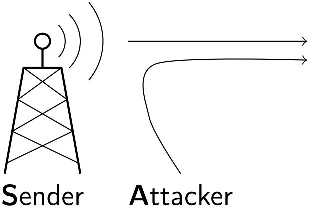

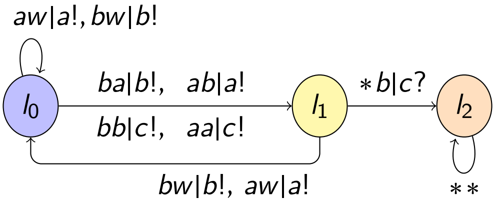

From $s =$ l $\cdot w$:

1. Pick an action for every player, then take the corresponding $l \xrightarrow{f} l'$
2. **Change** location to l' ;
3. Apply the **channel operation** f on w to get w';
4. Drop from w' to $w'' \preceq w'$: **subword ordering**.

Result: $s' =$ l' $\cdot w''$:

Contribution 1: Zero-sum case

The fixed point algorithms ([FS01, dAHK07]) still apply!

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NB: It is the "maximal" possible result:

-  Bertrand and al [BBS07] proves that $\mathrm{NZ}(\Box\Diamond R)$ (Büchi) and $\mathrm{AS}(\Box\Diamond R_1 \wedge \Diamond\Box R_2)$ cases are undecidable.
-  [May03] proved that $[\![\mathrm{E}(\Box R)]\!]_1$ cannot be computed.

Contribution 3: Core

Theorem

For a pair $(\mathscr{G}, \Gamma)$ *where players' objectives are almost-sure reachability or almost-sure safety objectives, and property* $\Gamma = \mathrm{AS}(\varphi)$ *with* φ *of the form* $\bigwedge_i \Diamond R_i$, $\bigwedge_i \Box R_i$, *or* $\bigwedge_i \Box\Diamond R_i$, *the problems of* E-Core *and* A-Core *are* <u>*decidable*</u>*.*

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- Guess the set of winning players $W \subseteq \mathrm{Agt}$;
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NB: "maximal" result since Büchi+coBüchi objectives make the problem undecidable [BBS07].

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Thank you for your attention

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Can't we just play with **DP** strategies only?

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P strategies **may** not be finitely represented. **PFM** are finitely represented, **Counting** too.

Strategy Classes, Updated, Counting

A strategy for player i is **C**ounting: if there exist two PFM strategies σ^u, σ^v such that:

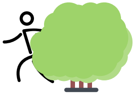
$$\forall n \geq 1, \forall h \in S^n, \sigma(h) = p_n \cdot \sigma^u(h) + (1 - p_n)\sigma^v(h)$$

Where:

$$p_n = 2^{-1/(2^k)}$$

Counting strategies are sufficient for winning $\mathrm{NZ}(\Box \cdots)$.

Skirmish Game [dAHK07]

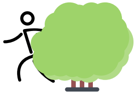


$A_1(s_0) = \{h, r\}$

$A_2(s_0) = \{s, w\}$

Played:

Skirmish Game [dAHK07]



$A_1(s_0) = \{h, r\}$ $A_2(s_0) = \{s, w\}$

Played: hw

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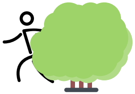

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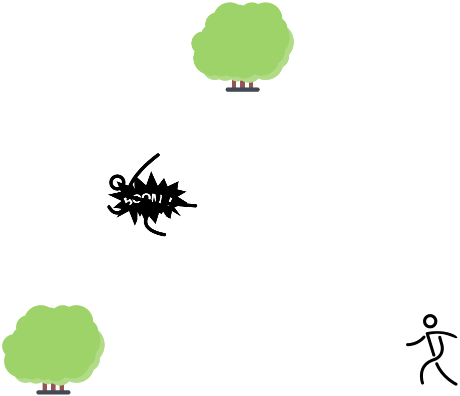

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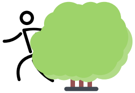

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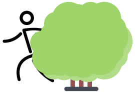

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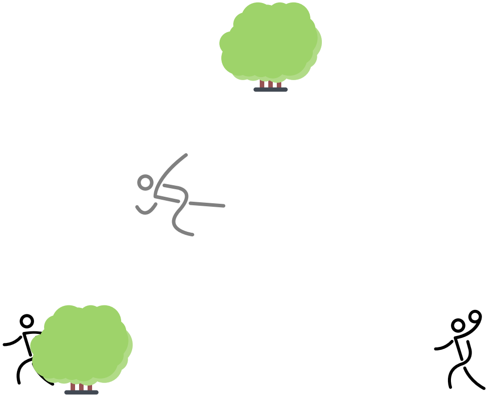

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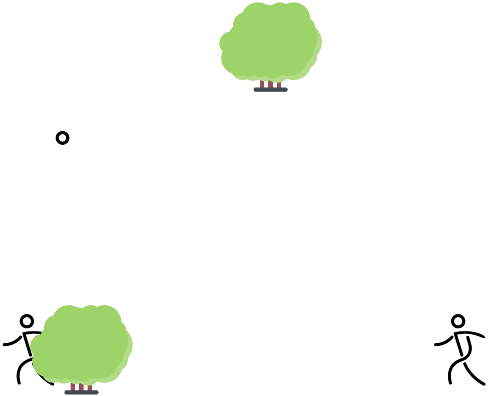

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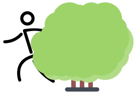

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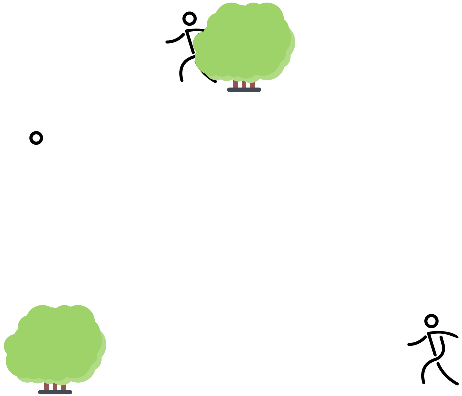

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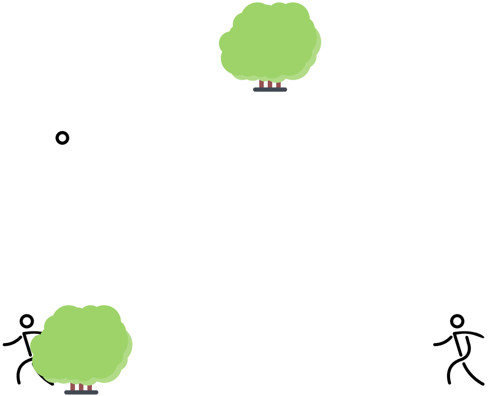

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